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Manifestation of singular features in black hole horizons and quasi-horizons

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Abstract

Usually, a black hole horizon hides singularities beyond it, being by itself perfectly regular. I discuss somewhat unusual cases when a system can combine regular and singular features due to the presence of the horizon. The first example is the so-called truly naked black holes when the Kretschmann scalar is finite but tidal forces diverge on the horizon in the free-falling frame. The second one is the so-called quasi-black holes (special kind of objects on the verge of forming the extremal black holes). Then, naked behavior can occur in the entire region beyond the quasi-horizon. Apart from this, there is another kind of singular behavior in this case due to the quasi-horizon. In particular, this concerns the simplest example with a charged massive shell, empty inside. The curvature inside is zero but the whole inner region becomes degenerate in the quasi-horizon limit.

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1. Introduction

In black hole physics, a horizon and singularity are implied to be entities of the opposite character. The usual picture consists of a regular horizon that hides singularities behind it. For example, in the Schwarzschild black hole there is a singularity at $r = 0$ whereas the horizon at $r = 2m$ (r is a curvature-like coordinate, m is a mass) is perfectly regular. The purpose of the present work is to pay attention that there are cases when the relationship between a horizon and regular or singular properties of the system (including the horizon itself) can be different and entangle in a rather non-trivial way. I discuss here two different although connected issues—the so-called 'truly naked' black holes and quasi-black holes. This discussion relies heavily on recent results [1, 2] where readers can find more detailed information.

2. Naked and truly naked black holes

Usually, a regular or singular character of points or surfaces in spacetime reveals itself as an inner property inherent to the manifold and does not depend on the frame in which it is described. In particular, the value of the Kretschmann scalar is finite or infinite, whatever frame is used for its calculation. Nonetheless, as was pointed out in [3, 4], in the vicinity of black holes in some cases the curvature components in the free-falling frame are enhanced significantly with respect to their static values to the extent that they are finite non-zero in spite of the fact that in the static frame they are negligible ('naked black holes'). Moreover, it is possible to reconcile the finiteness of the Kretschmann scalar with the divergences of tidal forces in the free-falling frame 'truly naked black holes' (TNBH) [5]. (Particular examples of this kind for an infinite area of the horizon were discussed in [6, 7].) This is connected with the Lorentz signature of the spacetime: in this frame different divergent terms cancel each other and the net outcome is finite.

Let us consider the spherically symmetric metric

$$ds^2 = -dt^2 U + V^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2). \tag{1}$$

Consider also the boosted frame moving in the radial direction with the boost angle α. Then the curvature components (hat stands for the orthonormal frame) are transformed according to

$$\hat{R}_{0'r'0'r'} = \hat{R}_{0r0r}, \qquad \hat{R}_{0'\theta'r'\theta'} = -\cosh\alpha \sinh\alpha \, Z, \qquad Z = R^{r\theta}_{r\theta} - R^{0\theta}_{0\theta}, \tag{2}$$

$$\hat{R}_{0'\theta'0'\theta'} = \hat{R}_{0\theta0\theta} + \sinh^2\alpha Z, \qquad \hat{R}_{r'\theta'r'\theta'} = \hat{R}_{r\theta r\theta} + \sinh^2\alpha Z \tag{3}$$

and similarly for components with θ replaced by ϕ. Here $\cosh\alpha = \frac{\varepsilon}{\sqrt{N}}$, ε is the energy per unit mass, N is the lapse function. (We choose $\alpha > 0$, then our definition differs from that in [3] by the sign.)

As the difference between the static and boosted frame reveals itself for all components (except $0r0r$ one) in a similar way, the analysis in [3] was mainly restricted to the component $R_{0\theta0\theta}$ that has a clear physical meaning, being responsible for tidal forces in the transverse directions. It is somewhat more convenient to deal with the combination of two components Z that includes the effect of tidal forces. From geodesics equations, one can obtain easily that the quantity Z is related to the energy density of the source $\bar{\rho}$ measured by a free-falling observer: $\bar{\rho} = T_{\mu\nu}u^\mu u^\nu = \frac{\varepsilon^2 Z}{4\pi U} - T_r^r\left(1 + \frac{L^2}{r^2}\right) + \frac{T_\phi^\phi L^2}{r^2}$, ε is the energy of a particle per unit mass along the geodesics, L is the angular momentum. It follows from the transformation laws (2), (3) that the quantity Z transforms as $\tilde{Z} = Z\left(2\frac{\varepsilon^2}{U} - 1\right)$. Thus,

$$\bar{\rho} = \frac{\bar{Z} + Z}{8\pi} - T_r^r\left(1 + \frac{L^2}{r^2}\right) + \frac{T_\phi^\phi L^2}{r^2}. \tag{4}$$

It follows from (4) that $\bar{\rho}$ diverges on the horizon if and only if when $\bar{Z}$ does so. In other words, on the horizon, $\bar{\rho}$ is infinite for TNBH and finite for usual and naked black holes.

We suppose that there is a horizon at $r = r_0$. We restrict ourselves by the simple asymptotics

$$V \approx a(r - r_0)^p, \qquad U \approx b(r - r_0)^q. \tag{5}$$

Then careful but direct analysis reveals the following features which can be summarized in the table. In doing so, we use the following definitions: (i) 'usual' ($Z \to 0, \tilde{Z} \to 0$), (ii) 'naked' ($Z \to 0, \tilde{Z} \to \text{const} \neq 0$), (iii) 'truly naked' ($Z \to 0, Z \to \infty$). It turns out that in cases $1 < p < \frac{3}{2}$, $p < q$, $q < p < q + 1$ TNBH are indeed possible.

Table 1. Types of horizons with finite area.

		Type of horizon
1	$p = q = 1$	Usual or naked
2	$1 < p < \frac{3}{2}$	Truly naked
3	$p = \frac{3}{2}$	Naked
4	$\frac{3}{2} < p < 2$	Usual
5	$p < q$	Truly naked
6	$p = q \geqslant 2$	Usual or naked
7	$q < p < q + 1$	Truly naked
8	$p = q + 1$	Naked
9	$q + 1 < p < q + 2$	Usual
10	$p \geqslant q + 2$	Usual

The existence of TNBH configurations discussed in the present paper as well as in the previous one [5] points to some potential rooms in scenarios of gravitational collapse which need further consideration. It also hints that the cosmic censorship should be somehow be reformulated to take into account these subtleties. It was pointed out in [3] that the existence of naked black hole may affect the issue of information loss and black hole entropy since large tidal forces significantly disturb the matter falling into a black hole. Even more so, this factor becomes important in the case of TNBH when tidal forces are not simply large but infinite on the horizon. There are three types of horizons in the aspect under discussion: 'usual' (in both frames curvature components are finite, tidal forces are zero), naked (in both frames curvature components are finite, tidal forces are zero for a static observer but finite non-vanishing for a free-falling observer), 'truly naked' (some curvature components are infinite for a free-falling observer).

3. Quasi-black holes

Usually, gravitational collapse is inevitable when the system boundary approaches the gravitational radius sufficiently closely. This is related to the Buchdahl limit [8], i.e. the minimum radius to mass ratio r_0/m that a stable configuration can have, where r_0 and m are the radius and mass of the configuration, respectively. For perfect fluid spheres it is $r_0/m \geqslant 9/4$, while for charged spheres the ratio decreases, it goes to $r_0/m \geqslant 1$ precisely in the case of extremal charged dust [9, 10]. However, it is remarkable that, contrary to the common case where instabilities set in much before a matter system reaches its own gravitational radius, there are some systems for which the gravitational radius can be approached in a sequence of static configurations. They are called quasi-black holes—QBHs (the term was coined in [11, 12]). Roughly speaking one can say that a QBH is an object on the verge of becoming an extremal BH but actually is distinct from it in many ways.

A QBH can be defined as an object with the following properties. Consider the static spherically symmetric metric, often written as

$$ds^2 = -U(r)\,dt^2 + V^{-1}(r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2), \tag{6}$$

where r is the Schwarzschild radial coordinate, and $U(r)$ and $V(r)$ are metric potentials. Let an inner matter configuration, with an asymptotic flat exterior region, exist with the properties (a) the function $V(r)$ attains a minimum at some $r^* \neq 0$, such that $V(r^*) = \varepsilon$, with $\varepsilon \ll 1$, this minimum being achieved either from both sides of r^* or from $r > r^*$ alone, (b) for such a small but non-zero ε the configuration is regular everywhere with a non-vanishing metric function U, at most the metric contains only delta-function like shells, and (c) in the limit $\varepsilon \to 0$ the metric coefficient $U \to 0$ for all $r \leqslant r^*$. These three features define a QBH. Note that although the above definition can be done in a form invariant under the choice of the radial coordinate since it is sufficient to replace V by $(\nabla r)^2$. In turn, these three features entail some non-trivial consequences: (i) there are infinite redshift whole regions, (ii) when $\varepsilon \to 0$, a free-falling observer finds in his own frame infinitely large tidal forces in the whole inner region, showing some form of degeneracy, although the spacetime curvature invariants remain perfectly regular everywhere, (iii) in the limit, outer and inner regions become mutually impenetrable and disjoint, and one can also show that (iv) for external far away observers the spacetime is virtually indistinguishable from that of extremal black holes. In addition, QBHs must be extremal. The QBH is on the verge of forming an event horizon, but it never forms one, instead, a quasihorizon appears. For a QBH the metric is well defined and everywhere regular. However, properties, such as when $\varepsilon = 0$, QBH spacetimes become degenerate, almost singular, have to be examined with care.

The physical examples of QBHs are the systems composed from the extremal dust (bounded or continuous distribution [13]), self-gravitating Higgs magnetic monopole systems [11, 12], composite spacetimes even in the case of pure electrovacuum [14].

Below we discuss the simplest case where gluing between an inner flat metric and an external extremal Reissner–Nordström metric is performed. Such a construction was discussed in [15] as an example of a classical model of an elementary particle (see also [16] and [14]). Consider an external spacetime given by equation (6) with $U = V = \left(1 - \frac{m}{r}\right)^2$ for $r \geqslant r_0$, and an inner spacetime given by the Minkowski metric,

$$ds^2 = -dT^2 + dr^2 + r^2\, d\Omega^2, \tag{7}$$

where $0 < r \leqslant r_0$. On the border, the condition of matching both parts of the spacetime leads to

$$t = \frac{T r_0}{r_0 - m}, \tag{8}$$

so that the time part of the metric (7) can be written as $-dT^2 = -\frac{(r_0-m)^2}{r_0^2}\, dt^2$. Then, if time t is used, the metric coefficient $g_{00} \to 0$ in the limit $r_0 \to m$. This is the reason why this construction can be considered as an example of a QBH. We again obtain an infinite redshift due to the mismatch in time rescaling in equation (8). Also, we cannot achieve the continuous matching if T is considered as a legitimate coordinate inside since the surface $r = m$ is timelike in the metric (7) but lightlike in the metric (6). One may try to repair this by considering inside the same time t as outside. However, the term $-\frac{(r_0-m)^2}{r_0^2}\, dt^2$ disappears in this limit and the spacetime becomes degenerate. If one calculates the surface stresses on the boundary, it turns out that $8\pi S_0^0 = -\frac{2}{m} \neq 0$ (all other components vanish) [14]. In this case, the formulae describing boost form a static observer to the free-falling one (cf equation (2), (3)) show that, again, a naked behavior takes place in the horizon limit when $r_0 \to m$.

Moreover, all QBHs share some common features, relevant in our context. Without going into details (see [2]), we enumerate some of them. Upon careful inspection, one finds that in QBHs divergencies on the Kretschmann scalar do not occur. However, the finiteness of this quantity is not the only criterion for regular or singular classification of a spacetime as we have seen while discussing TNBHs. In the present work, we have encountered a rather unusual entanglement of regular and singular features in QBHs. From the viewpoint of an external observer who uses time measured by clocks at infinity, an inner region of QBH looks like a degenerate spacetime with the component of the metric $g_{00} \to 0$ everywhere. Yet, this singular feature has nothing to do with the behavior of the Riemann tensor. Its components in an orthonormal static frame are finite there, and the Kretschmann scalar is also well behaved. The most obvious manifestation of this property is the example of the shell where the inner spacetime is flat, nonetheless it exhibits singular features! If one tries to remove the degeneracy of the inner spacetime by rescaling the time coordinate, another difficulty arises: the spacetime ceases to be continuous since the surface is lightlike from the viewpoint of an outer observer but is timelike from the viewpoint of an inner one. To put it in another way: one can easily achieve the validity of the matching conditions on a timelike surface, but if this surface tends to a null surface, at least from one side, the procedure ceases to be well defined and this gives rise to a number of unusual properties. As far as the components of the Riemann tensor are concerned, QBH may reveal a behavior typical of TNBH but not on the quasi-horizon only but in the whole region beyond it.

Another singular feature consists in the impossibility to penetrate from the inside to the outside and vice versa. In this sense, geodesics cannot be extended across the border between different regions, in spite of the fact that each of them, taken by itself, can be extended. For instance, the Minkowski spacetime in the example with the shell is obviously extendable but this extension has nothing to do with the problem under discussion in which the outer spacetime should be the extremal Reissner–Nordström BH. The fact that observers in different regions disagree about the border's nature, whether it is timelike or null, can be considered as one of the manifestations of the mutual impenetrability. Actually, it shows that one deals with two separate spacetimes. It turns out that there is some kind of complementary relationship between the inner and outer regions and between their regular and singular properties. If an observer is situated inside, he will say that the geometry is perfectly regular there but becomes singular on the border and beyond, so that he is unable to penetrate to outside. The outer observer, on the contrary, will say that it is his region which is regular (excepting the border) and finds he cannot penetrate into the inner singular region. All this forces us to conclude that the spacetime of a QBH as a whole may be singular in spite of the fact that the Kretschmann scalar diverges nowhere.

This discussion helps to elucidate an important additional question, of whether or not the QBH limit (whose properties we have discussed in detail) is attainable in some real physical process. For comparison, in the Reissner–Nordström geometry, taking formally the limit $q \to m$, one can obtain the extremal Reissner–Nordström BH from the nonextremal one but, according to the third law of BH thermodynamics, this cannot be accomplished in any real process for a finite number of steps. Furthermore, if the cosmic censorship conjecture is valid, one cannot convert the BH state with $q \leqslant m$ into a naked singularity by increasing the charge to $q > m$. What is said above about singular features in the QBHs properties leads to the conclusion that the corresponding limiting state is unattainable physically from any close regular configuration. More precisely, the state which is obtained by the mathematical procedure of taking the QBH limit can be approached as closely as one likes. However, if we assume that regular configurations cannot be turned into singular ones, the QBH limit cannot be attained by gradually changing an initial regular configurations to a singular one. An usual horizon hides singularities beyond it, but its analog, the quasihorizon, in a sense, brings about certain singular features into the system. Thus, we are faced with a somewhat unusual counterpart of the cosmic censorship. Nonetheless, in some other cases the limiting configuration may turn out to be geodesically complete and regular. In this case, nothing prevents one from taking the limit under discussion. Then, it seems that the limit can be attainable in some regions and unattainable in others, which is one more unusual feature of QBHs. On the face of what has been said, it seems that QBHs should extend the taxonomy, not only of relativistic objects, but also of singularity types in general relativity.

Summing up, configurations that approach as close as one likes a QBH state can be easily achieved, and in this sense, QBHs may have real physical significance. But whether a QBH state can be attained in nature, through such a process, or perhaps emerge via some quantum process, is a thorny issue that certainly needs further investigation.

4. Summary

Thus, we have seen that (i) a black hole horizon may look for a regular or singular observer depending on the frame, (ii) moreover, in the QBH case the presence of the quasi-horizon causes singular behavior of the system. Although we discussed the static configurations, these features should affect corresponding scenarios of gravitational collapse that needs further investigation.

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